# HOMEWORK 10

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ABSTRACT. Please send me an email if you find mistakes. Thanks.

## 1. P296. # 34.1

*Proof.* We follow the hint. Set $F(x) = \int_a^x g'$. Then

$$F'(x) = g'(x).$$

So

$$(F - g)' = 0.$$

Therefore $F(x) - g(x) = C$ for some constant $C$. Let $x = a$. Then

$$F(a) - g(a) = C.$$

Since $F(a) = \int_a^a g' = 0$,

$$C = -g(a).$$

Let $x = b$, we have

$$F(b) = \int_a^b g' = g(b) - g(a).$$

□

## 2. P296. # 34.2

*Proof.* **(a).** Let $F(x) = \int_0^x e^{t^2} dt$. Then

$$\lim \frac{1}{x} \int_0^x e^{t^2} dt = F'(0) = e^0 = 1.$$

**(b).** Let $F(x) = \int_0^x e^{t^2} dt$. Then

$$\lim_{h \to 0} \frac{1}{h} \int_3^{3+h} = \lim_{h \to 0} \frac{F(3+h) - F(3)}{h} = F'(3) = e^{3^2} = e^9.$$

□

## 3. P297. # 34.5

*Proof.* We write

$$G(x) = \int_0^x f(t)dt.$$

By Theorem 34.3, $G$ is differentiable on $\mathbb{R}$ and $G' = f$.

$$F(x) = \int_0^{x+1} f(t)dt - \int_0^{x-1} f(t)dt = G(x+1) - G(x-1).$$

Then $F$ is differentiable on $\mathbb{R}$ . Moreover by the chain rule,

$$F'(x) = f(x+1) - f(x-1).$$

□

## 4. P297. # 34.6

*Proof.* This is similar to the previous exercise.

$$G'(x) = f(\sin x)\cos x.$$

□

## 5. P297 # 34.7

*Proof.* Let $u = 1 - x^2$. Then $du = -2xdx$. So

$$\int_0^1 x\sqrt{1-x^2} = \int_0^1 \frac{\sqrt{u}}{2} du = \frac{1}{2} \times \frac{2}{3} u^{\frac{3}{2}}|_0^1 = \frac{1}{3}.$$

□

## 6. P297 # 34.11

*Proof.* We know that $f^2$ is continuous since $f$ is continuous. So by Theorem 33.4,

$$\int_a^b f^2(x)dx = 0$$

implies $f^2 = 0$. So $f \equiv 0$. □

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